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# A CRITICAL REVIEW OF THE ERROR POLYGON METHOD

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*Abstract:* The Error Polygon Method (EPM) of Heezen and Tester (1967) is used most frequently to quantify telemetry error. Complete scientific reporting for this method should include the confidence arcs with associated confidence levels, a measure of distance from the receiver to the estimated location, and a measure of the angle of intersection. The EPM assumption of a normal distribution of bearing errors was rejected with a large data base ( $n=940$ ) collected on 13 transmitter locations that resulted in 388 estimated locations. Empirical data of actual error demonstrated the EPM's ability to delineate 90% error polygons that contained the actual location >90% of the time, although both the area and longest diagonal of the error polygons were 6.3 and 7.4 times, respectively, larger than actual error. The EPM did not give an accurate measure of location error.

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The scientific method demands the reporting of accuracy and precision be complete so future research can be built on a solid base. Wildlife researchers commonly use radiotelemetry in the study of elusive or far-ranging species, such as bears. If telemetry is used to estimate an animal's location through triangulation, complete reporting of the error associated with such an estimated location is necessary.

There are 5 primary sources of telemetry error: system error, topographic error, reading error, movement error, and map error. System error is the result of inaccuracies inherent in the receiving system under standard field conditions. For example, once the receiving system is in the field, the directional sensitivity of its antennas can change through twisting by the wind, cable wear, or bent elements. Topographic errors arise from the radio signals being absorbed, deflected, or reflected by landscape components (Lee et al. 1985). Topographic errors are the most significant source of telemetry error, especially in mountainous terrain. Reading errors result from misreading the compass, the rosette bearing, or from incorrectly recording a bearing and can differ between observers. Reading errors are usually considered negligible or included in the angle error. Movement errors result when bearings on a moving animal are not taken simultaneously from different receiving stations. The time lag between bearings allows a radio-collared animal to move some distance, thereby giving single bearings on different actual locations. This type of error depends on the time lag and the activity of the animal between consecutive bearings (MacDonald and Amlaner 1980). Mech (1983) mentioned map error, which normally is not included in discussions of telemetry error. Plotting locations of receiving stations on maps will vary in accuracy and precision depending on the scale and accuracy of the maps used. An estimated location read from hand-plotted line bearings, as done in some field work, is influenced by the width of the marked lines and the placement of the compass rosette.

These sources of error affect angle error and location error. Springer (1979), MacDonald and Amlaner (1980), and Lee et al. (1985) described methods of data collection to minimize overall error. All locations should be on an x-y grid system (e.g., the Universal Transverse Mercator [UTM] system). Computer triangulation should be used to prevent increasing the effect of map error.

The most common method used to quantify radiotelemetry error is the Error Polygon Method (EPM). Heezen and Tester (1967) developed the concept of error polygons to allow improved locations for fixed receiving towers. They did not originally apply it to error associated with estimated locations, but to delineate a study area within which their estimates would have a known confidence. Springer (1979) discussed error polygons in depth. Currently, the EPM is used to report error in telemetry studies that often use the estimated location as a location without error in the subsequent analyses. The standard reporting of EPM includes a measure of bias and precision of bearings. Empirical data will show this is incomplete reporting.

I collected data on the actual error of several hundred estimated locations. I was then able to compare actual error to the predicted error of the EPM. My purpose was to investigate: 1) if an EPM 90% error polygon encompasses the actual location 90% of the time when applied to a comparable data set; and 2) if the 90% error polygon is effective in reflecting telemetry location error.

I appreciate the hours J.D. Hole, M.A. Horner, and E.O. Jones devoted to data collection and the support from my wife and son. Special thanks go to R.A. Powell and D.E. Seaman for their support and discussions of error.

## METHODS

### Data Collection

Data were collected during a black bear study conducted on the 220-km<sup>2</sup> Pisgah Black Bear Sanctuary in

the mountains of western North Carolina on the Pisgah National Forest. Transmitters and receivers (Telonics, Mesa, AZ) operated in the high frequency band. (Use of commercial name does not imply endorsement by North Carolina State University.) The receiving antenna was a truck-mounted, 2-m, 8-element directional antenna. Receiving stations were designated along the Blue Ridge Parkway (BRP) and several secondary roads. The BRP provided excellent access along the major ridge in the study area (Warburton 1984), allowing rapid bearing collection and good reception. Test data were collected from October 1983 through April 1985 on radio-collared bears in dens and on transmitters placed in the sanctuary. All bearings were recorded to the nearest degree using the 'loudest' signal technique (Springer 1979). Test bearings were taken during the observers' normally scheduled collection of data for estimating bear locations. Therefore, the error observed should be equivalent to the error associated with actual non-test data.

All test bearings were handled in the same manner as bear location data by initially hand-plotting on a U.S. Geological Survey map (scale 1:25,000). The plotting allowed a visual evaluation of each bearing in relation to other bearings and, more importantly, to topographic features. The visual evaluation determined: 1) if another bearing was needed and, if needed, which receiving station was most likely to provide a good bearing; and 2) which bearings were to be computer triangulated. All estimated locations were produced through triangulation on the UTM coordinate system by a computer program developed by the author. The computer program also calculated 2 distances associated with each estimate: 1) the location error - the distance between the estimated and actual location (the true location error); and 2) the geometric mean distance from the receiver to the estimated location for a single measure of distance.

The test data were randomly divided into 2 subsets, Analytic and Experimental. The Analytic data were used in the analyses of error. The results of these analyses were applied to the Experimental data to test the effectiveness of the EPM. Only locations estimated by 2 bearings with an angle of intersection between 45° and 135° were used in the Experimental data set. The reasons for this reduction in data were to standardize angles of intersection (Springer 1979) and to reduce computation complexities regarding the maintenance of a 90% error polygon confidence level.

A Duncan's multiple range comparison test for mean effect (Steele and Torrie 1980) was used to determine differences in bearing errors associated with the number of bearings used for the triangulation of an estimated

location and differences among the 5 observers. Two ratios were used to check the effectiveness of the EPM to indicate location error. The first ratio compared the area of the applied 90% error polygon to a circular error area where the radius was defined as the location error. The other ratio compared the longest diagonal of the applied 90% error polygon to twice the location error, both measurements reflecting a maximum error distance. All ratios <1 indicate the EPM error estimate was smaller than the actual error and ratios >1 indicate the EPM measure was larger than the actual error by a factor equal to the ratio.

### Error Polygon Method

The EPM gives a measure of location error using an indirect approach. The EPM treats angle error of each line bearing as the unit of analysis. The angle error of a system can be computed when exact locations are known for both receiving and transmitting stations:

$$\text{ANGLE ERROR} = \text{TRUE COMPASS BEARING} - \text{RECORDED BEARING}$$

Each angle error is statistically independent. Accuracy and precision of a telemetry system is estimated in the EPM by using the angle error. Angle accuracy has 2 components: bias and precision (Springer 1979, Lee et al. 1985, Saltz and Alkon 1985). Bias is a consistent error, reflects accuracy, and can be measured as the average angle error for a large error data set. Angle error can be positive or negative, indicating to which side of the true bearing the recorded bearing fell, clockwise or counterclockwise, respectively. The common measure of precision is the standard deviation of angle error for a large data set. The use of the standard deviation assumes normality. A comparable nonparametric measure for precision in a large data set is the interquartile range that encompasses the percentage of the angle errors equal to the desired confidence level. Each estimated bearing has an associated measure of precision. This precision can be envisioned as an error arc of some stated confidence level placed on each side of the estimated bearing to form a confidence arc. Two estimated bearings used for triangulation each have associated confidence arcs whose intersections form an error polygon (Fig. 1). Since the confidence arcs are independent of each other the confidence level of an error polygon produced by 2 95% confidence arcs equals 90% ( $0.95^2$ ). An error polygon has area and linear dimensions. The sizes and shapes of error polygons depend on 3 variables: confidence arcs, the angle of bearing intersection, and the distance from the receiver to the estimated transmitter location. Hupp and Ratti (1983) gave the following trigonometric rela-

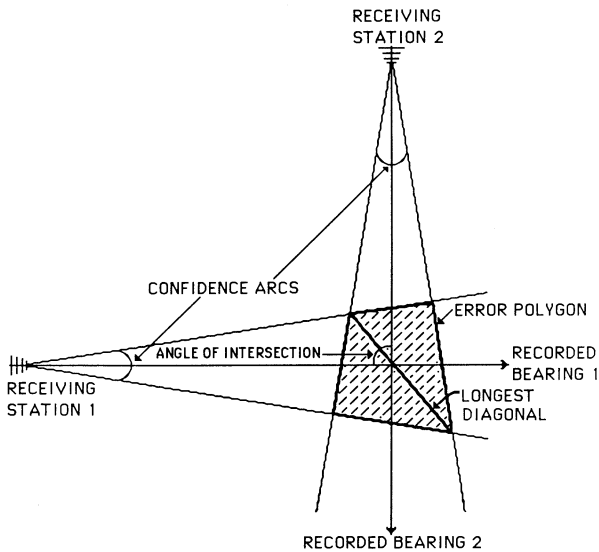


Fig. 1. The error polygon produced by 2 recorded bearings and their associated confidence arcs. The receiving stations are equal distance from the estimated location (point of bearing intersection). Recorded bearings intersect at 90°. The confidence of the error polygon equals the square of confidence associated to the arcs. The longest diagonal is the maximum error distance within the error polygon.

tionship of error polygon area for the special case of the angle of intersection equal to 90° and equal distances from the 2 receiving stations to the estimated location.

$$\text{ERROR POLYGON AREA (ha)} = \frac{(d^2 \sin 2\theta \tan 2\theta)}{(10,000 \text{ m/ha})}$$

where:  $d$  = distance (m) from receiver to estimated location;

$\theta$  = one side of the confidence arc.

This equation shows the area is a function of the square of the distance from the receiver to the estimated location. I revised this trigonometric equation to allow for any angle of intersection. (The computer algorithm is available on request from the author.) The revised equation was used to show the influence of the angle of intersection on error polygon area (Fig. 2).

Heezen and Tester (1967) used the longest diagonal within the error polygon to determine the outer limits of acceptable accuracy for fixed receiving towers. However, Saltz and Alkon (1985) suggested using the longest diagonal as a measure of accuracy for locations. This diagonal does give an estimate of error that is easy to interpret and includes the effects of distance and angle of intersection, although still based on confidence arcs and the bearing as the sample unit.

Several assumptions are made when using the EPM, 2 of which are necessary for stated confidences to be

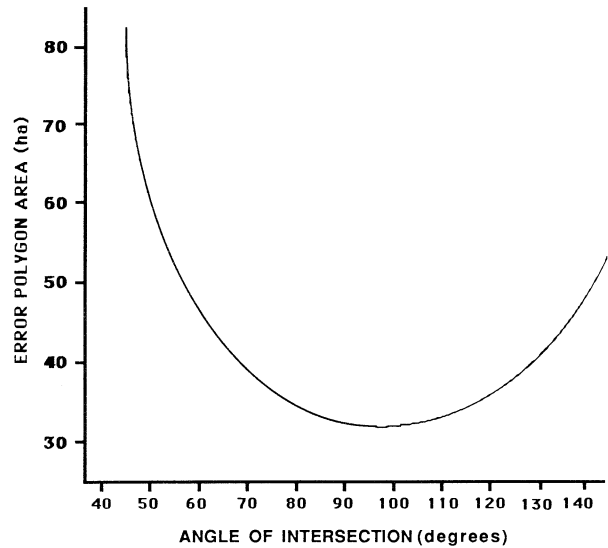


Fig. 2. The theoretical effect of the angle of intersection on area of a 90% error polygon. Distances from receiver to estimated locations and the confidence arcs are held constant at 1,000 m and ±16°, respectively.

correct. A normal distribution of angle errors and independence of angle errors are assumed.

## RESULTS

The test data contained 388 estimated locations from 940 bearings collected from 45 receiving stations on 13 locations. The resulting distribution of angle errors had a range from -39° to 71°. Although a histogram of angle errors appears normally distributed (Fig. 3), a Kolmogorov-Smirnov  $D$  test for normality (Steele and Torrie 1980) showed the distribution differed significantly from normal ( $D = 0.1057, P \leq 0.01$ ). The test data and the Analytic subset had a 95% confidence arc of ±16°, as indicated nonparametrically using a 95% interquartile range. The confidence arc was based on all bearings in the Analytic subset ( $n = 438$ ), regardless of the number of associated bearings. This confidence arc can be applied to the Experimental subset because a Duncan's multiple range test did not find any significant difference ( $\alpha = 0.05$ ) related to the number of bearings triangulated to estimate a location. No significant differences were found among the 5 observers (Duncan's multiple range test,  $\alpha = 0.05$ ).

A Sign rank test on the means found no bias in the test data ( $\bar{x} = 0.5^\circ, P < 0.25$ ) nor the Analytic subset ( $\bar{x} = 0.61^\circ, P < 0.29$ ). The geometric mean distance from receiver to estimated location ranged from 300 m to 6,020 m with a

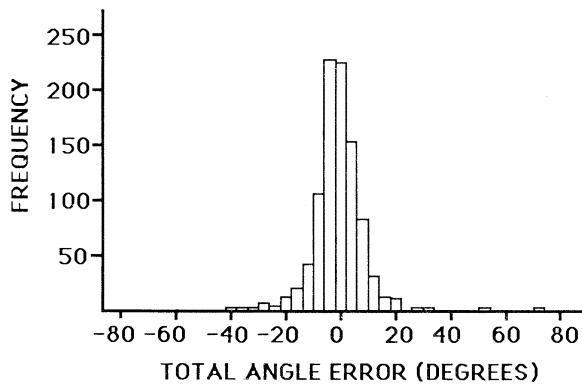


Fig. 3. Frequency of angle errors in the complete test data ( $n = 940$ ).

mean of 1,960 m. Angles of intersection ranged from  $7^\circ$  to  $167^\circ$  ( $\bar{x} = 68^\circ$ ).

The 95% confidence arc from the Analytic subset was applied to the Experimental subset to predict 90% error polygons. The Experimental data set contained 58 locations estimated from 106 bearings. Ninety-five percent of the estimated locations in the Experimental data were within their respective 90% error polygon. The mean area and longest diagonal of the error polygon were 235 ha and 3,150 m, respectively (Table 1). The location error averaged 310 m and the circular area averaged 49 ha. The ratio of 90% error polygon area to the location error circular area had a mean of 25.9, although a median of only 6.3. Ninety-four percent of the circular areas were smaller than the mean 90% error polygon area. The ratio of maximum distances demonstrated the longest diagonal averaged 7.4 times longer than twice the actual location error.

## DISCUSSION

The assumptions underlying the EPM are rarely mentioned in the literature. The major assumption of a normal distribution has not been challenged by a large test data set. The test data sets found in the literature are small, usually <100 bearings, and their size alone makes it hard to disprove an underlying distribution. Histograms have been used for a visual inspection of the data set checking for normality. This practice can be deceptive, as seen in this study. A researcher would actually hope for a highly peaked distribution centered around  $0^\circ$  with a few aberrant errors forming low tails, a Von Mises distribution with a large  $K$  (Mardia 1972). The independence of angle errors meets the statistical criteria for independence. However, the EPM cannot take into account that the 2 or more recorded bearings are toward a single location, nor does it assume to. A probable

influence of bearings from different receiving stations pointing to the same location and, therefore, influenced by similar topographic errors, cannot be handled in an application of the EPM. To test for this type of influence would be difficult.

The current reporting of the EPM error analyses usually includes the mean angle error (bias) and a 95% confidence arc (2SD), rarely indicating distances, with fewer reporting angles of intersection. This is incomplete reporting of telemetry error. The size and shape of the error polygon, hence the amount of error, depend not only on the size of the confidence arcs but on the squared distance from the receivers to the estimated location and the angle of bearing intersection (Fig. 2). Most reviews (Springer 1979, Lee et al. 1985) of the EPM state this but do not mention the need to report a measure of distance or angle of intersection. Without reporting the latter measures, no one can assess the effect of error on later analyses and conclusions. If I report no bias and a 95% confidence arc equal to  $\pm 16^\circ$ , no one can interpret the implications the error has on my results. To illustrate the point: Maintaining the  $\pm 16^\circ$  confidence arc and a  $90^\circ$  angle of intersection, while changing receiver distance from 0.5 km to 1.5 km, the area of the 90% error polygon increases 9-fold. Or, by maintaining the confidence arc and distance while changing the angle of intersection from  $90^\circ$  to  $65^\circ$ , the area of the error polygon increases 27% with a shape change. Therefore, for complete scientific reporting of the EPM analyses, a researcher needs to report bias, a measure of precision (95% confidence arcs), a measure of receiver to estimated location

Table 1. Descriptive statistics associated with the application of 95% confidence arcs to the Experimental test data set ( $n = 58$ ). Ratios were calculated to compare the EPM-indicated error to actual location error.

	$\bar{x}$ (SD)	Median	Range
Error Polygon Method			
90% confidence area (ha)	235 (233)	190	5 - 1,220
Longest diagonal (m)	3,150 (2,090)	3,000	360 - 9,250
Location Error			
Actual distance (m)	310 (245)	235	10 - 970
Circular error area (ha)	49 (71)	17	0 - 293
Ratios <sup>a</sup> of 90% Error Polygon to Location error			
Area ratio <sup>b</sup>	25.9 (45.4)	6.3	0.4 - 209.3
Diameter ratio <sup>c</sup>	7.4 (6.4)	4.8	1.1 - 26.9

<sup>a</sup> Ratios > 1 indicate the EPM measure was larger than the actual error by a factor equal to the ratio.

<sup>b</sup> Area of 90% error polygon relative to a circular area, where radius = location error.

<sup>c</sup> Longest diagonal of error polygon relative to twice the location error.

distance, and a measure of the angle of bearing intersection.

The effect of angle of intersection on polygon area is influenced by the confidence arc. My results from theoretical data showed the smallest area resulted from a 98° intersection (Fig. 2). This seemed to disagree with the reported best angle of 90° (Springer 1979), so I replaced my  $\pm 16^\circ$  confidence arc with  $\pm 1^\circ$ , which predicted the smallest area at a 90° intersection. Therefore, depending on the size of the confidence arc, the smallest error polygon area falls within a range of angles of intersection from 90° to 100°.

I found no bias in the accuracy of bearings, a common finding in reported EPM analysis. The precision in the current study, as indicated by the 95% confidence arc ( $\pm 16^\circ$ ), was similar to that found in other studies in forested, mountainous terrain. Edge and Marcum (1985) reported a 95% confidence arc of  $\pm 11.96^\circ$  ( $n = 79$ ) and Mooty et al. (1987) reported an absolute bearing error range of 0° to 40° ( $\bar{x} = 10^\circ$ ) at an average test distance of 614 m. My results were comparable to those of Mooty et al. (1987) despite a 3-fold increase in the mean test distance on this study ( $\bar{x} = 1,960$  m). The range of angles of intersection in this study was probably wider than for other studies. No measures of angles of intersection could be found in the literature; it is assumed most investigators follow Springer's (1979) recommendation of using only the bearings that intersect at angles between 45° and 135°.

This application of the EPM to a second set of field test data was the first of which I am aware. The predicted 90% error polygon contained at least 90% (actually 95%) of the true locations. These results showed clearly that the EPM worked, although the EPM does not effectively reflect location error. The size of predicted 90% error polygons averaged 235 ha, while the actual location error indicated a mean circular error area of 49 ha. In fact, the EPM so inaccurately reflected location error that 94% of the location error areas were less than the average 90% error polygon. The ratios of these 2 areas demonstrated that 5% of the true locations were outside the 90% error polygon. More importantly, the ratios proved the error polygons were 6.3 times larger than needed (using the median to indicate central tendency). The longest diagonal of the 90% error polygon indicated a location error for the data set of 3,150 m. Saltz and Alkon (1985) suggested its use because it represents the maximum error distance within an error polygon. This measure can be compared to the diameter of a circular area, where radius equals the location error, which averaged 620 m (2 radii). The ratios of maximum error distances showed the longest diagonal

reflected a location error 7.4 times larger than actual. The EPM does achieve a 90% error polygon, while consistently overestimating location error.

I recommend that researchers who still rely on the EPM always report bias, a confidence arc with the associated confidence levels, a measure of receiver to estimated location distance, and a measure of the angles of intersection. Alternatives to the EPM exist that provide statistically efficient estimators of the signal source and should be considered. Alternative methods are being developed to locate the source of an Emergency Locator Transmitter from downed aircraft. Lenth (1981) tested several methods (see Garrott et al. 1986) and Gutthorp and Lockhart (1988) developed another method. These alternate methods depend on microcomputer capabilities.

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